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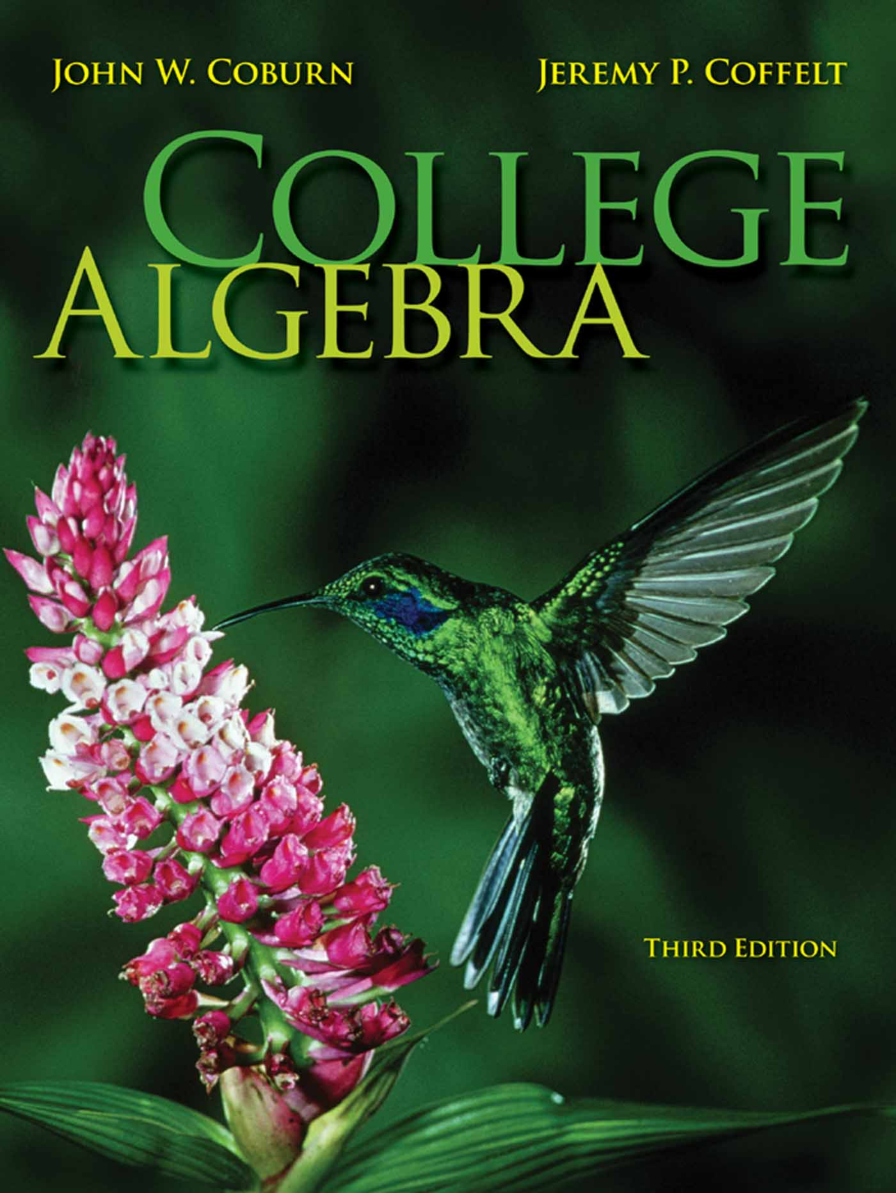

THIRD EDITION

COLLEGE ALGEBRA

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ST LOUIS COMMUNITY COLLEGE AT FLORISSANT VALLEY

JEREMY P. COFFELT
BLINN COLLEGE

THIRD EDITION

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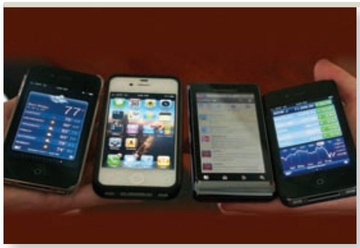

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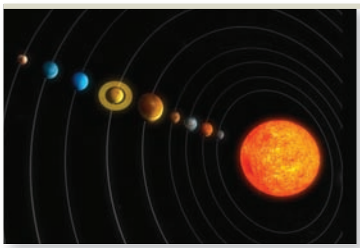

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About the Authors

John Coburn

John Coburn grew up in the Hawaiian Islands, the seventh of sixteen children. In 1977 he received his Associate of Arts Degree from Windward Community College, where he graduated with honors. In 1979 he earned a Bachelor's Degree in Education from the University of Hawai'i. After working in the business world for a number of years, he returned to teaching, accepting a position in high school mathematics, where he was recognized as Teacher of the Year in 1987. Soon afterward, a decision was made to seek a Master's Degree, which he received two years later from the University of Oklahoma. John is now a full professor at the Florissant Valley Campus of St. Louis Community College, where he has taught mathematics for the last twenty-one years. During this time he has received numerous nominations as an outstanding teacher by the local chapter of Phi Theta Kappa, earned recognition as a "Prime Time Teacher" by Eastern Illinois College in 2003, and was recognized as Post-Secondary Teacher of the Year in 2004 by Mathematics Educators of Greater St. Louis (MEGSL). John has made numerous presentations at local, state, and national conferences on a wide variety of topics, and maintains memberships in several mathematical organizations. Some of John's other interests include music, athletics, and the wild outdoors, as well as body surfing, snorkeling, and beach combing whenever he gets the chance. He is also an avid gamer, enjoying numerous board, card, and party games. John hopes that this love of life comes through in his writing, and helps to make the learning experience an interesting and engaging one for all students.

Jeremy Coffelt

Jeremy Coffelt grew up in the small town of Archer City, Texas, made (in)famous as the inspiration and filming location of *The Last Picture Show*. After graduating from Archer City High School in 2000, he continued his education at Midwestern State University, where he graduated with a Bachelor's Degree in Mathematics in 2002. From there, he completed Master's Degrees in Mathematics from Kansas State University (2005) and in Civil Engineering from Texas A&M University (2008). During his graduate studies, Jeremy published several papers in topics ranging from analytic number theory to Bayesian regression and engineering systems reliability. In 2007, he joined the faculty at Blinn College, where he has since been nominated for several teaching awards. When not teaching or writing, Jeremy enjoys spending time with his ladies—his wife Vanessa, his Chihuahua Buttons, and his Catahoula Abby. His other interests include traveling and all things competitive, including cycling, pool, chess, poker, and tennis.

Dedication

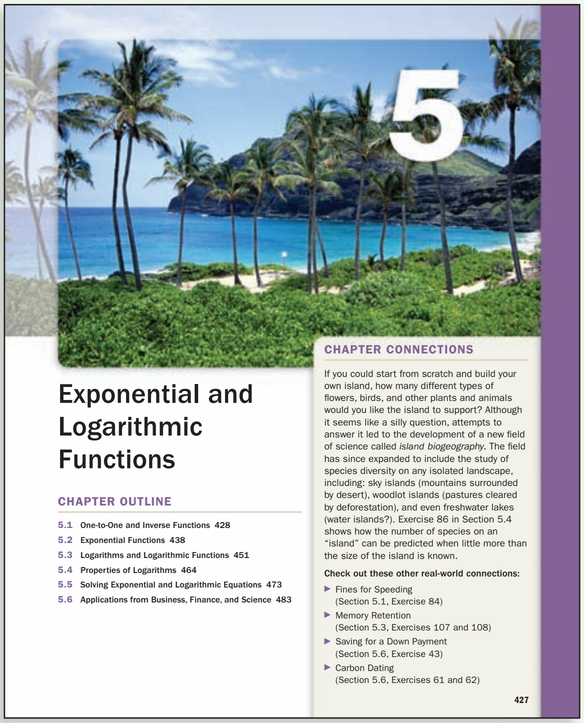
I dedicate this work to each of my seven children, in hopes it will help them discover a love of mathematics from their father, as I discovered a love of mathematics from my own. *John Coburn*

I dedicate my contributions to this text to my wife, Vanessa. For the times you left me alone to work, I thank you. For the times you interrupted my work, I love you. *Jeremy Coffelt*

Making Connections

A Focus on Applications

- **Chapter Openers** highlight Chapter Connections, an interesting application exercise from the chapter, and provide a list of other real-world connections to give context for students who wonder how math relates to them.
- **Application Exercises** at the end of each section are the hallmark of the Coburn-Coffelt series. Never contrived, always creative, and borne out of the authors' lives and experiences, each application tells a story and appeals to a variety of teaching styles, disciplines, backgrounds, and interests. The authors have ensured that the applications reflect the most common majors of college algebra students.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Exponential and Logarithmic Functions

CHAPTER OUTLINE

5.1 One-to-One and Inverse Functions 428
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5.4 Properties of Logarithms 464
5.5 Solving Exponential and Logarithmic Equations 473
5.6 Applications from Business, Finance, and Science 483

CHAPTER CONNECTIONS

If you could start from scratch and build your own island, how many different types of flowers, birds, and other plants and animals would you like the island to support? Although it seems like a silly question, attempts to answer it led to the development of a new field of science called *island biogeography*. The field has since expanded to include the study of species diversity on any isolated landscape, including: sky islands (mountains surrounded by desert), woodlot islands (pastures cleared by deforestation), and even freshwater lakes (water islands?). Exercise 86 in Section 5.4 shows how the number of species on an "island" can be predicted when little more than the size of the island is known.

Check out these other real-world connections:

- Fines for Speeding (Section 5.1, Exercise 84)
- Memory Retention (Section 5.3, Exercises 107 and 108)
- Saving for a Down Payment (Section 5.6, Exercise 43)
- Carbon Dating (Section 5.6, Exercises 61 and 62)

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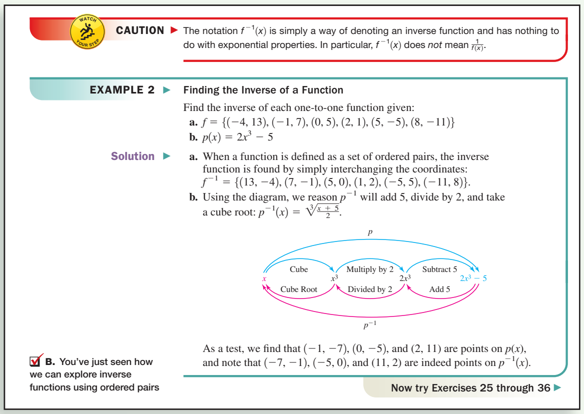
Clear and Timely Examples

- **Examples** are designed with a direct focus on the skill at hand while linking to previous concepts and laying the groundwork for concepts to come. The examples provide students with a starting point for solving a variety of problems.
- **Caution Boxes** signal students to pause so that they avoid common errors.
- **Check Points** alert students when a specific learning objective has been covered to reinforce correct mathematical terms.
- **"Now Try"** boxes immediately following examples guide students to specific matched exercises at the end of the section and connect concepts to homework problems.
- **Graphical Examples** show students how the calculator can be used to enhance their understanding.


CAUTION ► The notation $f^{-1}(x)$ is simply a way of denoting an inverse function and has nothing to do with exponential properties. In particular, $f^{-1}(x)$ does *not* mean $\frac{1}{f(x)}$.

EXAMPLE 2 ► Finding the Inverse of a Function

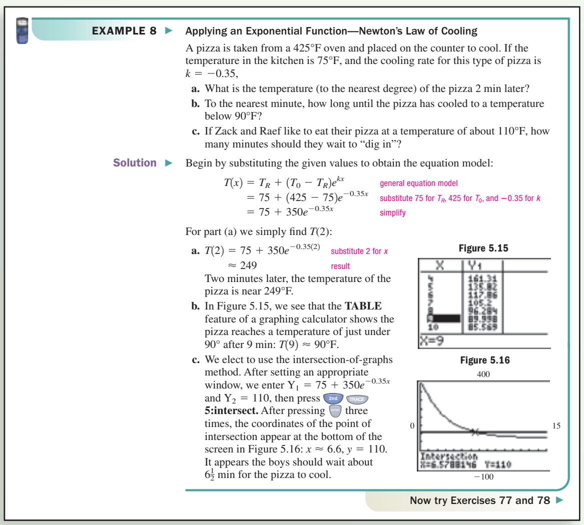
Find the inverse of each one-to-one function given:
a. $f = \{(-4, 13), (-1, 7), (0, 5), (2, 1), (5, -5), (8, -11)\}$
b. $p(x) = 2x^3 - 5$

Solution ► a. When a function is defined as a set of ordered pairs, the inverse function is found by simply interchanging the coordinates: $f^{-1} = \{(13, -4), (7, -1), (5, 0), (1, 2), (-5, 5), (-11, 8)\}$.
b. Using the diagram, we reason p^{-1} will add 5, divide by 2, and take a cube root: $p^{-1}(x) = \sqrt[3]{\frac{x+5}{2}}$.

As a test, we find that $(-1, -7)$, $(0, -5)$, and $(2, 11)$ are points on $p(x)$, and note that $(-7, -1)$, $(-5, 0)$, and $(11, 2)$ are indeed points on $p^{-1}(x)$.

B. You've just seen how we can explore inverse functions using ordered pairs

Now try Exercises 25 through 36 ►


EXAMPLE 8 ► Applying an Exponential Function—Newton's Law of Cooling

A pizza is taken from a 425°F oven and placed on the counter to cool. If the temperature in the kitchen is 75°F, and the cooling rate for this type of pizza is $k = -0.35$,
a. What is the temperature (to the nearest degree) of the pizza 2 min later?
b. To the nearest minute, how long until the pizza has cooled to a temperature below 90°F?
c. If Zack and Raef like to eat their pizza at a temperature of about 110°F, how many minutes should they wait to "dig in"?

Solution ► Begin by substituting the given values to obtain the equation model:

$$T(x) = T_R + (T_0 - T_R)e^{kx}$$ general equation model
$$= 75 + (425 - 75)e^{-0.35x}$$ substitute 75 for T_R, 425 for T_0, and −0.35 for k
$$= 75 + 350e^{-0.35x}$$ simplify

For part (a) we simply find $T(2)$:

a. $T(2) = 75 + 350e^{-0.35(2)}$ substitute 2 for x
≈ 249 result
Two minutes later, the temperature of the pizza is near 249°F.

b. In Figure 5.15, we see that the **TABLE** feature of a graphing calculator shows the pizza reaches a temperature of just under 90° after 9 min: $T(9) \approx 90$°F.

c. We elect to use the intersection-of-graphs method. After setting an appropriate window, we enter $Y_1 = 75 + 350e^{-0.35x}$ and $Y_2 = 110$, then press 2nd TRACE **5:intersect.** After pressing ENTER three times, the coordinates of the point of intersection appear at the bottom of the screen in Figure 5.16: $x \approx 6.6$, $y = 110$. It appears the boys should wait about $6\frac{1}{2}$ min for the pizza to cool.

Figure 5.15

X	Y1
4	161.31
5	135.82
6	117.86
7	105.2
8	96.284
9	89.998
10	85.569

X=9

Figure 5.16

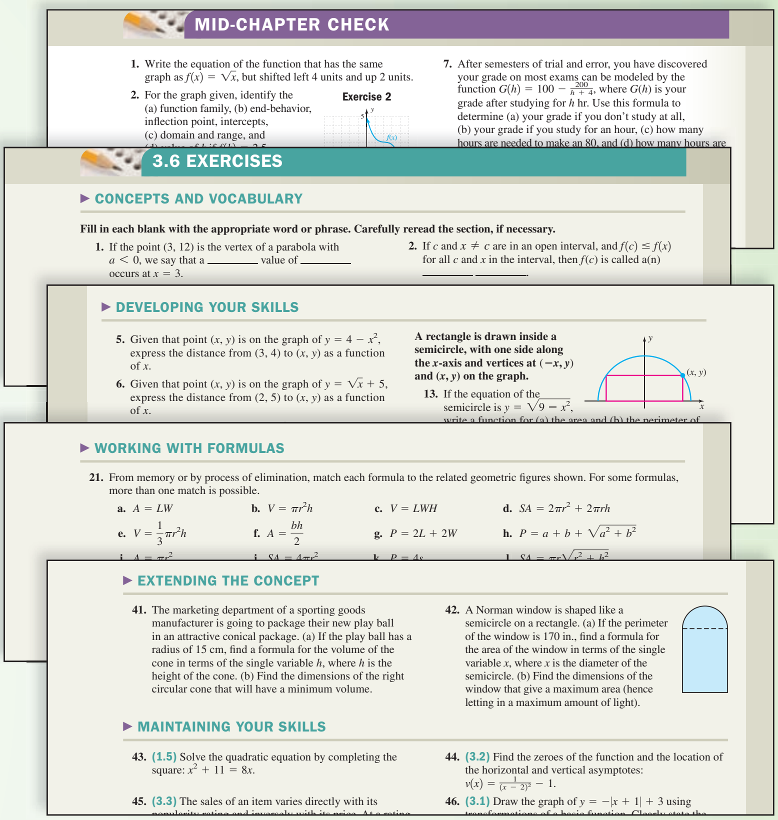
Now try Exercises 77 and 78 ►

Comprehensive Exercise Sets

- **Mid-Chapter Checks** provide students with a sampling of exercises to assess their knowledge before moving on to the second half of the chapter.

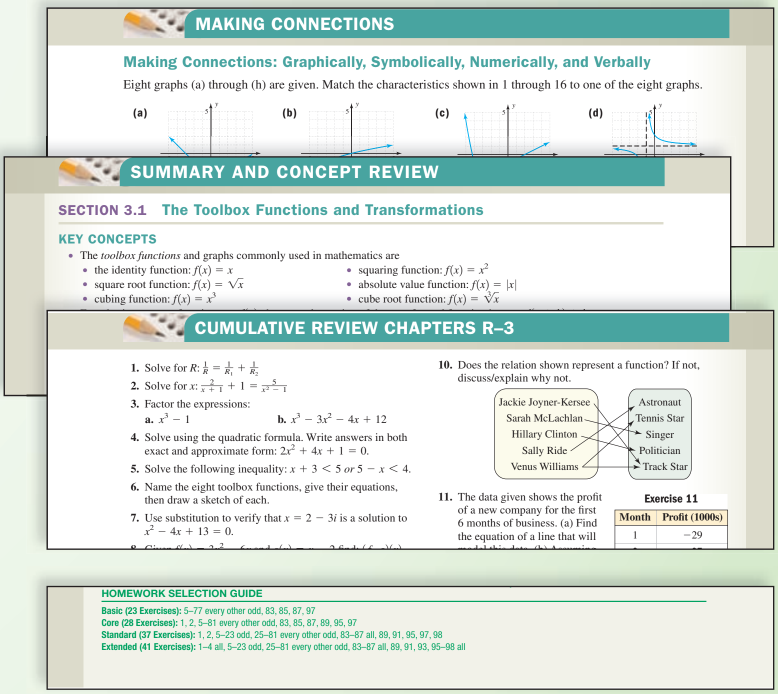
End-of-Section Exercise Sets

- **Concepts and Vocabulary** exercises help students recall and articulate important terms.
- **Developing Your Skills** exercises provide students with practice of essential concepts with increasing levels of difficulty.
- **Working with Formulas** exercises demonstrate contextual applications of well-known formulas.
- **Extending the Concept** exercises challenge students to extend their knowledge and skills.
- **Maintaining Your Skills** exercises address skills from previous sections to help students retain knowledge after learning new concepts.



End-of-Chapter Review Material

- **Making Connections** are matching exercises that help students interpret graphical and algebraic information.
- **Chapter Summary and Concept Reviews** present key concepts by section and are paired with corresponding exercises.
- **Practice Tests** enable students to check their knowledge and prepare for assessments.
- **Cumulative Reviews,** at the end of each chapter, revisit important concepts from earlier chapters so that students can retain their skills.
- **Graphing Calculator** icons appear next to exercises where concepts can be supported by graphing technology.
- **Homework Selection Guide** A list of suggested homework exercises has been provided for each section of the text (Annotated Instructor's Edition only). The guide provides preselected assignments at four levels: *Basic, Core, Standard,* and *Extended*.



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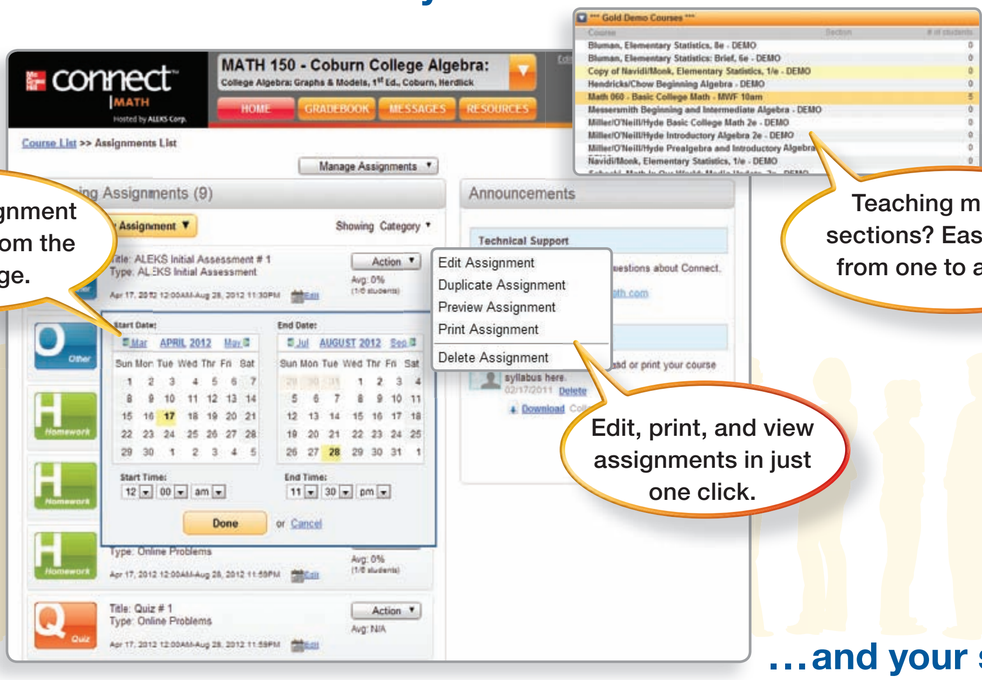

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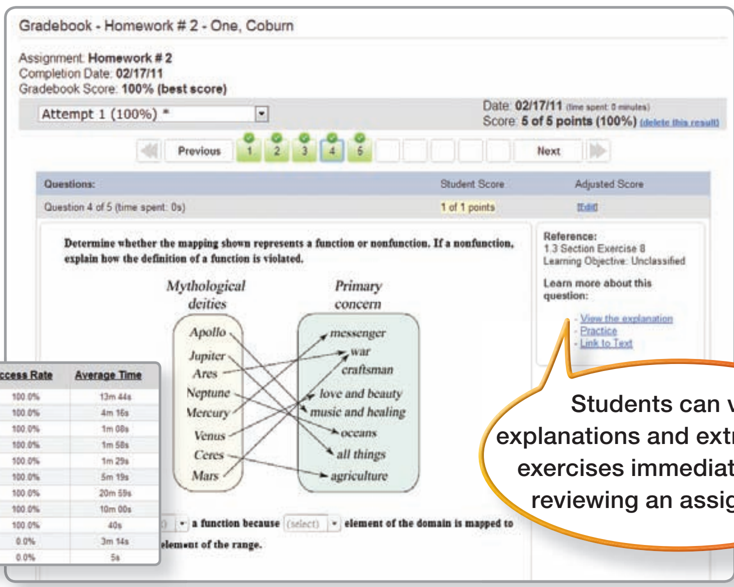
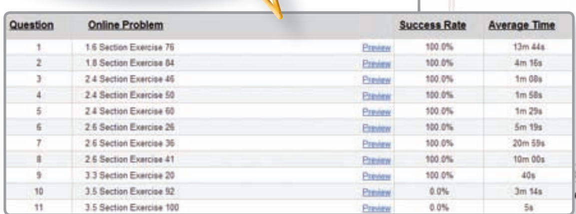


Know exactly where your students are struggling and how much time they're spending on each topic.



Question	Online Problem		Success Rate	Average Time
1	1.6 Section Exercise 76	Preview	100.0%	13m 44s
2	1.8 Section Exercise 84	Preview	100.0%	4m 16s
3	2.4 Section Exercise 46	Preview	100.0%	1m 08s
4	2.4 Section Exercise 50	Preview	100.0%	1m 58s
5	2.4 Section Exercise 60	Preview	100.0%	1m 29s
6	2.6 Section Exercise 26	Preview	100.0%	5m 19s
7	2.6 Section Exercise 36	Preview	100.0%	20m 59s
8	2.6 Section Exercise 41	Preview	100.0%	10m 00s
9	3.3 Section Exercise 20	Preview	100.0%	40s
10	3.5 Section Exercise 92	Preview	0.0%	3m 14s
11	3.5 Section Exercise 100	Preview	0.0%	5s

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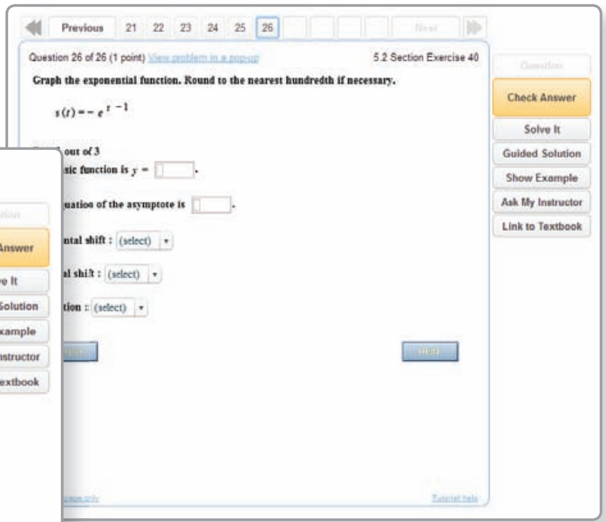

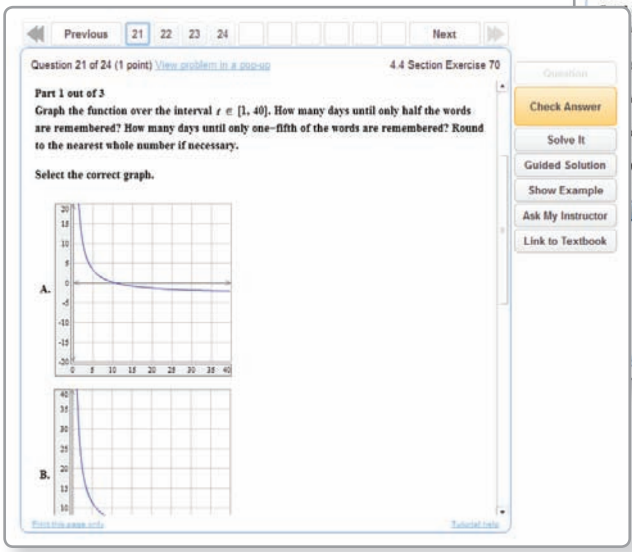

For consistency, the guided solutions match the style and voice of the original text, as though the author is guiding the students through the problems.

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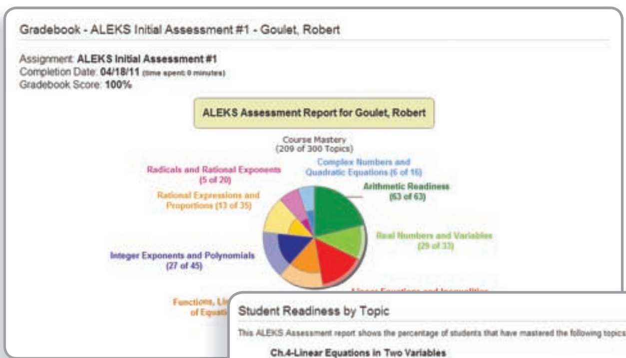

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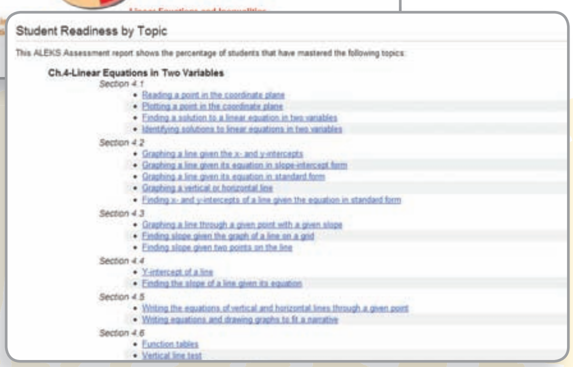

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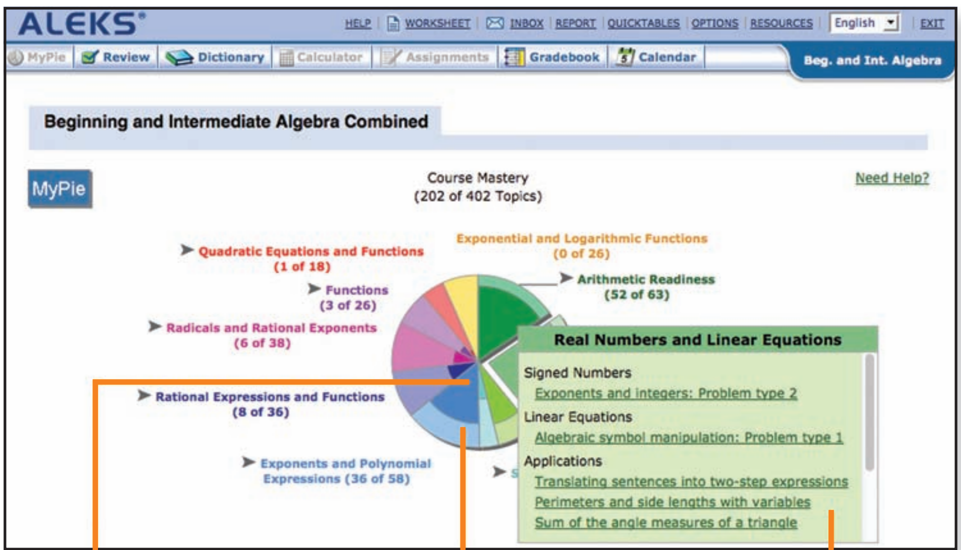

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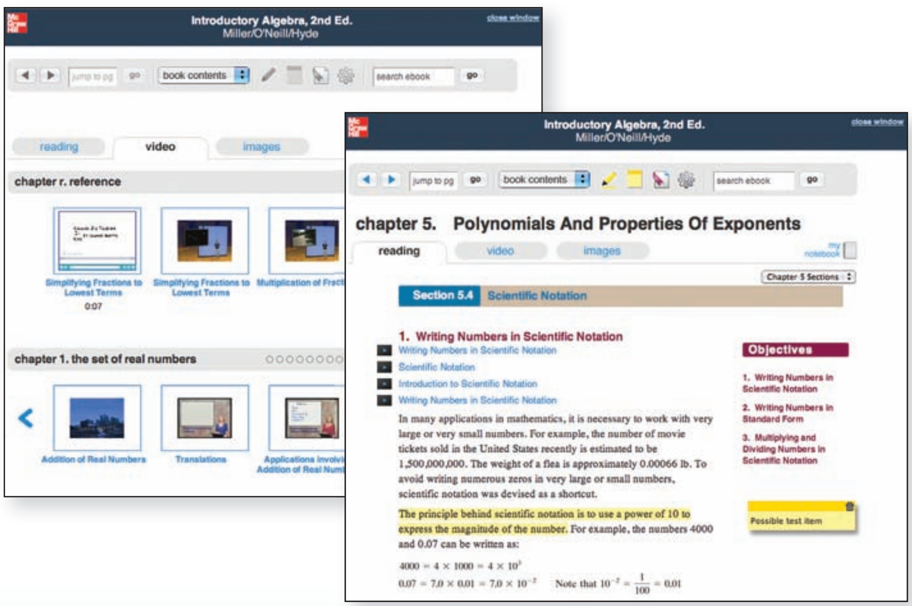

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Acknowledgments and Reviewers

We first want to express a deep appreciation for the guidance, comments, and suggestions offered by all reviewers. We have once again found their collegial exchange of ideas and experience very refreshing, instructive, and always helpful in creating a better learning tool for our students. Then there's Vicki Krug, who has continued to display an uncanny ability to bring innumerable pieces from all directions into a unified whole. With Patricia Steele's skill as a copy editor being as sharp as ever, her attention to detail continues to pay great dividends. For their useful suggestions, infinite patience, and tireless efforts in bringing this text to completion, we would also like to thank Ryan Blankenship, Caroline Celano, and Ashley Zellmer. We are especially grateful to Ashley for holding the enterprise together as the winds of change buffeted us once again. We must also thank Laurie Janssen and our magnificent design team, and Emilie Berglund as the Director of Digital Content, for helping shape a text that John Osgood, Kevin Ernzen, Tim Cote, Pete Vanaria, and our entire sales force can really get behind. In truth, our hats are off to all the fine people at McGraw-Hill for their continuing support and belief in this series. Additional gratitude must go to Hal Whipple for his attention to detail during the accuracy checks; Jon Booze and his team for their work on the test banks; J.D. Herdlick, Laura Taylor, Kara Allara, and Cathy Gonzalez for their contributions to the lecture and exercise video creation; with special thanks to Vanessa Coffelt, Stephen Toner, Amy Naughten, and Michael Lanstrum for working so hard to make Connect® Math Hosted by ALEKS the superb homework system it has become. A final word of thanks is due Rick Armstrong, Anne Marie Mosher, Rick Pescarino, Kelly Ballard, John Elliot, Jim Frost, Barb Kurt, Lillian Seese, Nate Wilson, and all of our colleagues at St. Louis Community College and Blinn College, whose friendship, encouragement, and love of mathematics makes going to work each day a joy.

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Student Solutions Manual The *Student Solutions Manual* provides comprehensive, worked-out solutions to all of the odd-numbered exercises in the text.

Lecture Notes provide a lecture template for students to fill-in important topics, terms, and procedures so they spend less time creating notes and more time engaged in class.

Computerized Test Bank (CTB) Online The computerized test bank contains a variety of question types, including true-false, multiple-choice, short-answer. The Brownstone Diploma® system enables you to efficiently select, add, and organize questions by type or level of difficulty. It also allows for printing tests complete with answer keys, as well as editing existing questions or creating new ones. Printable tests and a print version of the test bank can also be found on the website.

Lecture and Exercise Videos Lecture videos introduce concepts, definitions, formulas, and problem-solving procedures to help students better comprehend the topic at hand. Exercise videos provide step-by-step instruction for the key exercises which students will most wish to see worked out. The videos are closed-captioned for the hearing-impaired, are subtitled in Spanish, and meet the Americans with Disabilities Act Standards for Accessible Design.

Instructor Solutions Manual The *Instructor Solutions Manual* provides comprehensive, worked-out solutions to all exercises. The steps shown in the solutions match the style of the textbook.

Annotated Instructor's Edition (instructors only) The Annotated Instructor's Edition contains answers to all exercises. The answers to most questions are printed in a second color next to each problem. Answers not appearing on the page can be found in the Answer Appendix at the end of the book.

List of Changes to the Third Edition

- **More than a third of the examples** are new or revised and **nearly 1000 exercises** are new or revised in this edition.
- All chapters now have six sections or less—a tremendous aid to testing, review, coverage, and retention.
- Exercise instructions have been shortened and made easier to follow and understand.
- Exercises Sets have been reorganized according to difficulty in order to build student understanding.
- Most Applications have been carefully reviewed, improved, and updated.
- Most Multipart exercises have been broken down to clarify what's being asked and how a student should answer.
- Most Applications involving rates of interest have been modified to more nearly match interest rates of the day.
- **New Chapter Openers and *Chapter Connections*** have been added to all chapters to reflect modern issues and ideas.
- In **Chapter R, A Review of Basic Concepts,** sections R.4, R.5, and R.6 have been reordered so that radical expressions follow exponents.
- In **Chapter 2, Relations, Functions and Graphs,** coverage of topics including slopes, rates of change, and the difference quotient have been expanded.
- **New Section 2.6, Linear Models and Real Data,** is focused on real-world situations where data is best modeled by a linear function.
- **New Chapter 3, More on Functions,** builds on the concepts from Chapter 2, expanding a student's understanding of functions by introducing the algebra of functions and additional families of functions, including basic rational and power functions, and piecewise-defined functions.
- **Chapter 4, Polynomial and Rational Functions,** now boasts a more streamlined coverage of rational functions, with a greater focus on the most important characteristics of rational graphs (zeroes and asymptotic behavior). Coverage of complex solutions to polynomial equations has been regrouped to offer instructors better coverage options.
- **Chapter 5, Exponential and Logarithmic Functions,** has been restructured to include additional emphasis on logarithmic properties, the change of base formula, and real-world applications.

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R

A Review of Basic Concepts and Skills

CHAPTER OUTLINE

CHAPTER CONNECTIONS

Participation in many common recreational activities depends on the time of year, or even on the time of day. For instance, we expect that attendance at state parks will be greater in the spring than in the winter, and that a swimming pool will have more swimmers at 1:00 P.M. (in the heat of the day) than at 8:00 A.M. The equation $S = -h^2 + 10h$ can be used to estimate number of people in a swimming pool at any time of day, where S is the number of swimmers and h is the number of hours the pool has been open. This chapter reviews the skills required to estimate the number of swimmers in the pool at a given time of day, as well as other mathematical skills to be used throughout the course. This equation appears as Exercise 128 in Section R.3.

Check out these other real-world connections:

- Pediatric Dosages and Clark's Rule (Section R.1, Exercise 104)
- Maximizing Revenue of Video Game Sales (Section R.3, Exercise 129)
- Accident Investigation (Section R.4, Exercise 55)
- Growth of a New Stock Hitting the Market (Section R.6, Exercise 75)

R.1 The Language, Notation, and Numbers of Mathematics

LEARNING OBJECTIVES

In Section R.1 you will review how to:

- ☐ **A.** Identify sets of numbers, graph real numbers, and use set notation
- ☐ **B.** Use inequality symbols and order relations
- ☐ **C.** Use the absolute value of a real number
- ☐ **D.** Apply the order of operations

The most fundamental requirement for learning algebra is mastering the words, symbols, and numbers used to express mathematical ideas. "Words are the symbols of knowledge, the keys to accurate learning" (Norman Lewis in *Word Power Made Easy*, Penguin Books).

A. Sets of Numbers, Graphing Real Numbers, and Set Notation

To effectively use mathematics as a problem-solving tool, we must first be familiar with the **sets of numbers** used to quantify (give a numeric value to) the things we investigate. Only then can we make comparisons and develop equations that lead to informed decisions.

Natural Numbers

The most basic numbers are those used to count physical objects: 1, 2, 3, 4, and so on. These are called **natural numbers** and are represented by the capital letter $\mathbb{N}$, often written in the special font shown. We use **set notation** to list or describe a set of numbers. Braces { } are used to group **members** or **elements** of the set, commas separate each member, and three dots (called an *ellipsis*) are used to indicate a pattern that continues indefinitely. The notation $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$ is read, "$\mathbb{N}$ is the set of numbers 1, 2, 3, 4, 5, and so on." To show membership in a set, the symbol $\in$ is used. It is read "is an element of" or "belongs to." The statements $6 \in \mathbb{N}$ (6 is an element of $\mathbb{N}$) and $0 \notin \mathbb{N}$ (0 is not an element of $\mathbb{N}$) are true statements. A set having no elements is called the **empty** or **null set,** and is designated by empty braces { } or the symbol $\varnothing$.

EXAMPLE 1 ► **Writing Sets of Numbers Using Set Notation**

List the set of natural numbers that are

a. greater than 100 **b.** negative

c. greater than or equal to 5 and less than 12

Solution ►

a. $\{101, 102, 103, 104, \ldots\}$

b. { }; all natural numbers are positive.

c. $\{5, 6, 7, 8, 9, 10, 11\}$

Now try Exercises 5 and 6 ►

Whole Numbers

Combining zero with the natural numbers produces a new set called the **whole numbers** $\mathbb{W} = \{0, 1, 2, 3, 4, \ldots\}$. We say that the natural numbers are a **proper subset** of the whole numbers, denoted $\mathbb{N} \subset \mathbb{W}$, since every natural number is also a whole number. The symbol $\subset$ means "is a proper subset of."

EXAMPLE 2 ► **Determining Membership in a Set**

Given $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4\}$, and $C = \{0, 1, 2, 3, 5, 8\}$, determine whether the following statements are true or false. Justify your response.

a. $B \subset A$ **b.** $B \subset C$ **c.** $C \subset \mathbb{W}$

d. $C \subset \mathbb{N}$ **e.** $104 \in \mathbb{W}$ **f.** $2 \notin \mathbb{W}$

Solution ▶

a. True: Every element of B is in A.

b. False: $4 \notin C$, so $B \not\subseteq C$.

c. True: All elements are whole numbers.

d. False: $0 \notin \mathbb{N}$, so $C \not\subseteq \mathbb{N}$.

e. True: 104 is a whole number.

f. False: 2 *is* a whole number.

Now try Exercises 7 through 12 ▶

Integers

Numbers greater than zero are **positive numbers.** Every positive number has an *opposite* that is a **negative number** (a number less than zero). Combining zero with the natural numbers and their opposites produces the set of **integers** $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$. We can illustrate the location and magnitude of a number (in relation to other numbers) using a **number line** (see Figure R.1).

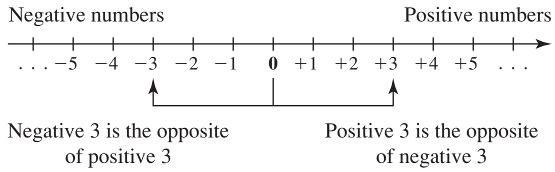


Figure R.1

The number that corresponds to a given point on the number line is called the **coordinate** of that point. When we want to note a specific location on the line, a bold dot "•" is used and we have then **graphed** the number. Since we need only one coordinate to denote a location on the number line, it is referred to as a **one-dimensional graph.**

> **WORTHY OF NOTE**
>
> The integers are a subset of the rational numbers: $\mathbb{Z} \subset \mathbb{Q}$, since any integer can be written as a fraction using a denominator of one: $-2 = \frac{-2}{1}$ and $0 = \frac{0}{1}$, and the like.

Rational Numbers

Fractions and mixed numbers are part of a set called the **rational numbers** $\mathbb{Q}$. A rational number is one that can be written as a fraction with an integer numerator and an integer denominator other than zero. In set notation we write $\mathbb{Q} = \{\frac{p}{q} | p, q \in \mathbb{Z}; q \neq 0\}$. The vertical bar "|" is read "such that" and indicates that a description follows. In words, we say, "$\mathbb{Q}$ is the set of numbers of the form p over q, such that p and q are integers and q is not equal to zero."

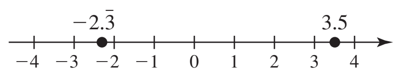
EXAMPLE 3 ▶ Graphing Rational Numbers

Graph the fractions by converting to decimal form and estimating their location between two integers:

a. $-2\frac{1}{3}$ **b.** $\frac{7}{2}$

Solution ▶

a. $-2\frac{1}{3} = -2.3333333\ldots$ or $-2.\overline{3}$ **b.** $\frac{7}{2} = 3.5$



Now try Exercises 13 through 16 ▶

Since the division $\frac{7}{2}$ **terminated,** the result is called a **terminating decimal.** The decimal form of $-2\frac{1}{3}$ is called **repeating** and **nonterminating** (note that $-2.3 \neq -2.\overline{3}$). Recall that a repeating decimal is written with a horizontal bar over the first block of digit(s) that repeat. For instance $\frac{118}{55} = 2.1454545\ldots = 2.1\overline{45}$.

When using a calculator for computations involving repeating decimals, you must either use the rational form or "fill the display" with the digits that repeat. As an exploration, suppose that you are to inherit $\frac{1}{3} = 0.\overline{3}$ of a \$90,000 estate. How many "repeating threes" (times 90,000) are needed until the calculator returns an answer of \$30,000? **See Exercises 17 and 18.**

WORTHY OF NOTE

Checking the approximation for $\sqrt{5}$ shown, we obtain $2.2360679^2 = 4.999999653$. While we can find better approximations by using more and more decimal places, we never obtain five *exactly* (although some calculators will say the result is 5 due to limitations in programming).

Irrational Numbers

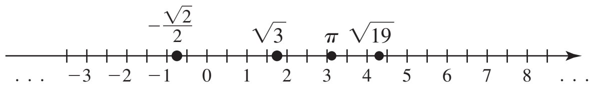
Although any fraction can be written in decimal form, not all decimal numbers can be written as a fraction. One example is the number represented by the Greek letter π (pi), frequently seen in a study of circles. Although we often approximate π using 3.14, its true value has a **nonrepeating** and *nonterminating* decimal form. Other numbers of this type include 2.101001000100001 . . . (there is no block of digits that repeat—the number of zeroes between each "1" is increasing), and $\sqrt{5} \approx 2.2360679\ldots$ (the decimal form never terminates). Numbers with a nonrepeating and nonterminating decimal form belong to the set of irrational numbers $\mathbb{H}$.

EXAMPLE 4 ▶ Approximating Irrational Numbers

Use a calculator as needed to approximate the value of each number given (round to 100ths), then graph them on the number line:

a. $\sqrt{3}$ **b.** π **c.** $\sqrt{19}$ **d.** $-\frac{\sqrt{2}}{2}$

Solution ▶ **a.** $\sqrt{3} \approx 1.73$ **b.** $\pi \approx 3.14$ **c.** $\sqrt{19} \approx 4.36$ **d.** $-\frac{\sqrt{2}}{2} \approx -0.71$



Now try Exercises 19 through 22 ▶

Real Numbers

The set of rational numbers combined with the set of irrational numbers produces the set of **real numbers** $\mathbb{R}$. Figure R.2 illustrates the relationship between the sets of numbers we've discussed so far. Notice how each subset appears "nested" in a larger set.

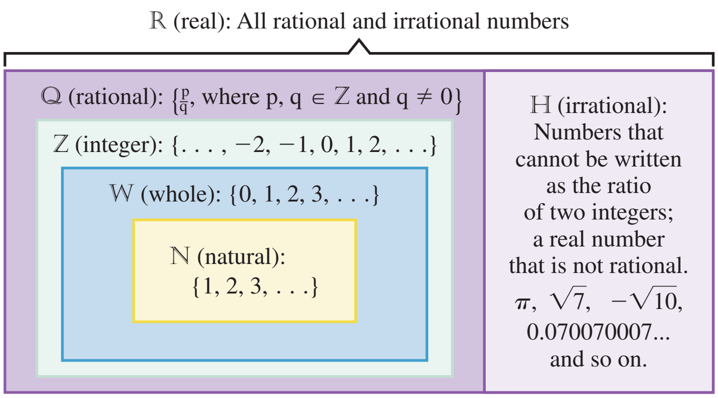


Figure R.2

EXAMPLE 5 ▶ Identifying Members of a Number Set

List the numbers in set $A = \{-2, 0, 5, \sqrt{7}, 12, \frac{2}{3}, 4.5, \sqrt{21}, \pi, -0.75\}$ that belong to

a. $\mathbb{Q}$ **b.** $\mathbb{H}$ **c.** $\mathbb{W}$ **d.** $\mathbb{Z}$

Solution ▶ **a.** $-2, 0, 5, 12, \frac{2}{3}, 4.5, -0.75 \in \mathbb{Q}$ **b.** $\sqrt{7}, \sqrt{21}, \pi \in \mathbb{H}$

c. $0, 5, 12 \in \mathbb{W}$ **d.** $-2, 0, 5, 12 \in \mathbb{Z}$

Now try Exercises 23 through 26 ▶

EXAMPLE 6 ▶ **Evaluating Statements about Sets of Numbers**

Determine whether the statements are true or false. Justify your response.

a. $\mathbb{N} \subset \mathbb{Q}$ **b.** $\mathbb{H} \subset \mathbb{Q}$ **c.** $\mathbb{W} \subset \mathbb{Z}$ **d.** $\mathbb{Z} \subset \mathbb{R}$

Solution ▶

a. True: All natural numbers can be written as fractions over 1.
b. False: No irrational number can be written in fraction form.
c. True: All whole numbers are integers.
d. True: Every integer is a real number.

A. You've just reviewed how to identify sets of numbers, graph real numbers, and use set notation

Now try Exercises 27 through 38 ▶

B. Inequality Symbols and Order Relations

We compare numbers of different size using **inequality notation,** known as the **greater than** (>) and **less than** (<) symbols. Note that $-4 < 3$ is the same as saying -4 is to the left of 3 on the number line. In fact, on a number line, any given number is smaller than any number to the right of it (see Figure R.3).

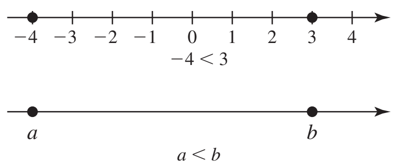


Figure R.3

Order Property of Real Numbers

Given any two real numbers a and b.

1. $a < b$ if a is to the left of b on the number line.
2. $a > b$ if a is to the right of b on the number line.

Inequality notation is used with numbers and variables to write mathematical statements. A **variable** is a symbol, commonly a letter of the alphabet, used to represent an unknown quantity. Over the years x, y, and n have become most common, although any letter (or symbol) can be used. Often we'll use variables that remind us of the quantities they represent, like L for length, and D for distance.

EXAMPLE 7 ▶ **Writing Mathematical Models Using Inequalities**

Use a variable and an inequality symbol to represent the statement: "To hit a home run out of Jacobi Park, the ball must travel over three hundred twenty-five feet."

Solution ▶ Let D represent distance: $D > 325$ ft.

Now try Exercises 39 through 42 ▶

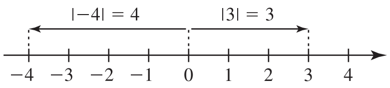
In Example 7, note the number 325 itself is not a possible value for D. If the ball traveled *exactly* 325 ft, it would hit the fence and stay in play. Numbers that mark the limit or boundary of an inequality are called **endpoints.** If the endpoint(s) are *not* included, the less than (<) or greater than (>) symbols are used. When the endpoints *are* included, the *less than or equal to symbol* (≤) or the *greater than or equal to symbol* (≥) is used. The decision to *include* or *exclude* an endpoint is often an important one, and many mathematical decisions (and real-life decisions) depend on a clear understanding of the distinction. **See Exercises 43 through 48.**

B. You've just reviewed how to use inequality symbols and order relations

C. The Absolute Value of a Real Number

Any nonzero real number "n" is either a positive number or a negative number. But in some applications, our primary interest is simply the *size* of n, rather than its sign. This is called the **absolute value** of n, denoted $|n|$, and can be thought of as its *distance from zero on the number line*, regardless of the direction (see Figure R.4). Since distance is always positive or zero, $|n| \geq 0$.

Figure R.4



EXAMPLE 8 ▶ Absolute Value Reading and Reasoning

In the table shown, the absolute value of a number is given in column 1. Complete the remaining columns.

Solution ▶

Column 1 (In Symbols)	Column 2 (Spoken)	Column 3 (Result)	Column 4 (Reason)
$\|7.5\|$	"the absolute value of seven and five-tenths"	7.5	the distance between 7.5 and 0 is 7.5 units
$\|-2\|$	"the absolute value of negative two"	2	the distance between −2 and 0 is 2 units
$-\|-6\|$	"the opposite of the absolute value of negative six"	−6	the distance between −6 and 0 is 6 units, the opposite of 6 is −6

Now try Exercises 49 through 56 ▶

Example 8 illustrates that the absolute value of a positive number is the number itself, while the absolute value of a negative number is the *opposite of that number* (recall that $-n$ is positive if n itself is negative). For this reason, the formal definition of absolute value is stated as follows.

Absolute Value

For any real number n,

$$|n| = \begin{cases} n & \text{if } n \geq 0 \\ -n & \text{if } n < 0 \end{cases}$$

The concept of absolute value can actually be used to find the distance between *any* two numbers on a number line. For instance, we know the distance between 2 and 8 is 6 (by counting). Using absolute values, we can write $|8 - 2| = |6| = 6$, or $|2 - 8| = |-6| = 6$. Generally, if a and b are two numbers on the real number line, the distance between them is $|a - b|$, which is identical to $|b - a|$.

EXAMPLE 9 ▶ Using Absolute Value to Find the Distance between Points

Find the distance between −5 and 3 on the number line.

Solution ▶ Substituting −5 for a and 3 for b in the formula shown gives

$$|-5 - 3| = |-8| = 8 \quad \text{or} \quad |3 - (-5)| = |8| = 8.$$

Now try Exercises 57 through 64 ▶

☑ **C.** You've just reviewed how to use the absolute value of a real number

D. The Order of Operations

The operations of addition, subtraction, multiplication, and division are defined for the set of real numbers, and the concept of absolute value plays an important role. Prior to our study of the order of operations, we will review fundamental concepts related to division and zero, exponential notation, and square roots/cube roots.

Division and Zero

The quotient $\frac{36}{9} = 4$ can be checked using the related multiplication: $4 \cdot 9 = 36$✓. A similar check can be used to understand quotients involving zero.

EXAMPLE 10 ▶ Understanding Division with Zero by Writing the Related Product

Rewrite each quotient *using the related product.*

a. $0 \div 8 = p$ **b.** $\frac{16}{0} = q$ **c.** $\frac{0}{12} = n$

Solution ▶

a. $0 \div 8 = p$, if $p \cdot 8 = 0$. This shows $p = 0$.

b. $\frac{16}{0} = q$, if $q \cdot 0 = 16$. There is no such number q.

c. $\frac{0}{12} = n$, if $n \cdot 12 = 0$. This shows $n = 0$.

Now try Exercises 65 through 68 ▶

WORTHY OF NOTE

When a pizza is delivered to your home, it often has "8 parts to the whole," and in fraction form we have $\frac{8}{8}$. When all 8 pieces are eaten, 0 pieces remain and the fraction form becomes $\frac{0}{8} = 0$. However, the expression $\frac{8}{0}$ is meaningless (undefined), since it would indicate a pizza that has "0 parts to the whole (??)."

In Example 10(a), a dividend of 0 and a divisor of 8 means we are going to divide zero into eight groups. The related multiplication shows there will be zero in each group. As in Example 10(b), an expression with a divisor of 0 *cannot be computed or checked*. Although it seems trivial, division by zero has many implications in a study of mathematics, so make an effort to know the facts: The quotient of zero and any nonzero number is zero ($\frac{0}{n} = 0$) but division *by* zero is undefined ($\frac{n}{0}$ is undefined). The special case of $\frac{0}{0}$ is said to be **indeterminate,** as $\frac{0}{0} = n$ *appears* to be true for all real numbers n (since the check gives $n \cdot 0 = 0$✓). The expression $\frac{0}{0}$ is studied in greater detail in more advanced classes.

Division and Zero

The quotient of zero and any nonzero number n is zero ($n \neq 0$):

$$0 \div n = 0 \qquad \frac{0}{n} = 0.$$

The expressions $n \div 0$ and $\frac{n}{0}$ are undefined.

Squares, Cubes, and Exponential Form

When a number is repeatedly multiplied by itself as in (10)(10)(10)(10), we write it using **exponential notation** as 10^4. The number used for repeated multiplication (in this case 10) is called the **base,** and the superscript number is called an **exponent.** The exponent tells how many times the base occurs as a factor, and we say 10^4 is written in **exponential form.** Numbers that result from squaring an integer are called **perfect squares,** while numbers that result from cubing an integer are called **perfect cubes.** These are often collected into a table, such as Table R.1, and students

Table R.1

Perfect Squares			
N	N^2	N	N^2
1	1	7	49
2	4	8	64
3	9	9	81
4	16	10	100
5	25	11	121
6	36	12	144

Perfect Cubes	
N	N^3
1	1
2	8
3	27
4	64
5	125
6	216

are strongly encouraged to memorize these values to help complete many common calculations mentally. Only the square and cube of selected positive integers are shown.

EXAMPLE 11 ▶ **Evaluating Numbers in Exponential Form**

Write each exponential in expanded form, then determine its value.

a. 4^3 **b.** $(-6)^2$ **c.** -6^2 **d.** $\left(\frac{2}{3}\right)^3$

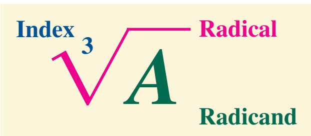
Solution ▶ **a.** $4^3 = 4 \cdot 4 \cdot 4 = 64$ **b.** $(-6)^2 = (-6) \cdot (-6) = 36$

c. $-6^2 = -(6 \cdot 6) = -36$ **d.** $\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$

Now try Exercises 69 and 70 ▶

Examples 11(b) and 11(c) illustrate an important distinction. The expression $(-6)^2$ gives a single operation, "the square of negative six" and the negative sign is included in both factors. The expression -6^2 gives two operations, "six is squared, and the result is made negative." The square of six is calculated first, with the negative sign applied afterward.

Square Roots and Cube Roots



For the square root operation, either the $\sqrt{\ }$ or $\sqrt[2]{\ }$ notation can be used. The $\sqrt{\ }$ symbol is called a **radical,** the number under the radical is called the **radicand,** and the small number used is called the **index** (see figure). The index tells how many factors are needed to obtain the radicand. For example, $\sqrt{25} = 5$, since $5 \cdot 5 = 5^2 = 25$ (when the $\sqrt{\ }$ symbol is used, the index is understood to be 2). In general, $\sqrt{a} = b$ only if $b^2 = a$. All numbers greater than zero have one positive and one negative square root. The *positive* or **principal square root** of 49 is 7 $(\sqrt{49} = 7)$ since $7^2 = 49$. The *negative* square root of 49 is -7 $(-\sqrt{49} = -7)$.

The cube root of a number has the form $\sqrt[3]{a} = b$, where $b^3 = a$. This means $\sqrt[3]{27} = 3$ since $3^3 = 27$, and $\sqrt[3]{-8} = -2$ since $(-2)^3 = -8$. The cube root of a real number has one unique real value. In general, we have the following:

WORTHY OF NOTE

It is helpful to note that both 0 and 1 are their own square root, cube root, and *n*th root. That is, $\sqrt{0} = 0$, $\sqrt[3]{0} = 0, \ldots, \sqrt[n]{0} = 0$; and $\sqrt{1} = 1$, $\sqrt[3]{1} = 1, \ldots, \sqrt[n]{1} = 1$.

Square Roots	Cube Roots
For $a \geq 0$, $\sqrt{a} = b$ if $b^2 = a$.	For $a \in \mathbb{R}$, $\sqrt[3]{a} = b$ if $b^3 = a$.
This indicates that	This indicates that
$\sqrt{a} \cdot \sqrt{a} = a$ or $(\sqrt{a})^2 = a$	$\sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a$ or $(\sqrt[3]{a})^3 = a$

EXAMPLE 12 ▶ **Evaluating Square Roots and Cube Roots**

Determine the value of each expression.

a. $\sqrt{49}$ **b.** $\sqrt[3]{125}$ **c.** $\sqrt{\frac{9}{16}}$ **d.** $-\sqrt{16}$ **e.** $\sqrt{-25}$

Solution ▶ **a.** $\sqrt{49} = 7$ since $7 \cdot 7 = 49$ **b.** $\sqrt[3]{125} = 5$ since $5 \cdot 5 \cdot 5 = 125$

c. $\sqrt{\frac{9}{16}} = \frac{3}{4}$ since $\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$ **d.** $-\sqrt{16} = -4$ since $\sqrt{16} = 4$

e. $\sqrt{-25}$ is not a real number [note that $5 \cdot 5 = (-5)(-5) = 25$].

Now try Exercises 71 through 76 ▶

WORTHY OF NOTE

Sometimes the acronym **PEMDAS** is used as a more concise way to recall the order of operations: **P**arentheses, **E**xponents, **M**ultiplication, **D**ivision, **A**ddition, and **S**ubtraction. The idea has merit, so long as you remember that multiplication and division *have an equal rank,* as do addition and subtraction, and these must be computed in the order they occur (from left to right).

For square roots, if the radicand is a perfect square or has perfect squares in both the numerator and denominator, the result is a rational number as in Examples 12(a) and 12(c). If the radicand is not a perfect square, the result is an irrational number. Similar statements can be made regarding cube roots [Example 12(b)].

The Order of Operations

When basic operations are combined into a larger mathematical expression, we use a specified **priority** or **order of operations** to evaluate them.

The Order of Operations

1. Simplify within grouping symbols (parentheses, brackets, braces, etc.). If there are "nested" symbols of grouping, begin with the innermost group. If a fraction bar is used, simplify the numerator and denominator separately.
2. Evaluate all exponents and roots.
3. Compute all multiplications or divisions *in the order they occur from left to right.*
4. Compute all additions or subtractions *in the order they occur from left to right.*

EXAMPLE 13 ▶ **Evaluating Expressions Using the Order of Operations**

Simplify using the order of operations:

a. $5 + 2 \cdot 3$

b. $8 + 36 \div 4(12 - 3^2)$

c. $\dfrac{-4.5(8) - 3}{\sqrt[3]{125} + 2^3}$

d. $7500\left(1 + \dfrac{0.075}{12}\right)^{12 \cdot 15}$

Solution ▶

a. $5 + 2 \cdot 3 = 5 + 6$ — multiplication before addition

$= 11$ — result

b. $8 + 36 \div 4 \cdot (12 - 3^2)$

$= 8 + 36 \div 4 \cdot (12 - 9)$ — simplify within parentheses

$= 8 + 36 \div 4 \cdot (3)$ — $12 - 9 = 3$

$= 8 + 9(3)$ — the division occurs first

$= 8 + 27$ — multiply

$= 35$ — result

c. $\dfrac{-4.5(8) - 3}{\sqrt[3]{125} + 2^3}$ — original expression

$= \dfrac{-36 - 3}{5 + 8}$ — simplify terms in the numerator and denominator

$= \dfrac{-39}{13}$ — combine terms

$= -3$ — result

d. $7500\left(1 + \dfrac{0.075}{12}\right)^{12 \cdot 15}$ — original expression

$= 7500(1.00625)^{12 \cdot 15}$ — simplify within the parenthesis (division before addition)

$= 7500(1.00625)^{180}$ — simplify the exponent so it can be applied

$\approx 7500(3.069451727)$ — exponents before multiplication

$\approx 23{,}020.88795$ — result

WORTHY OF NOTE

Many common tendencies are hard to overcome. For instance, let's evaluate the expressions $3 + 4 \cdot 5$ and $24 \div 6 \cdot 2$. For the first, the correct result is 23 (multiplication before addition), though some will get 35 by adding first. For the second, the correct result is 8 (multiplication or division *in order*), though some will get 2 by multiplying first.

Now try Exercises 77 through 102 ▶

D. You've just reviewed how to apply the order of operations